

On a general formalism of nonlinear charge coherent states, their quantum statistics and nonclassical properties

F. Eftekhari and M. K. Tavassoly

Atomic and Molecular Group, Faculty of Physics, Yazd University, Yazd, Iran

e-mail: mktavassoly@yazduni.ac.ir

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Abstract

In this paper, we will present a general formalism for constructing the nonlinear charge coherent states which in special case lead to the standard charge coherent states. The $su_Q(1,1)$ algebra as a nonlinear deformed algebra realization of the introduced states is established. In addition, the corresponding even and odd nonlinear charge coherent states have been also introduced. The formalism has the potentiality to be applied to systems either with known "nonlinearity function" $f(n)$ or solvable quantum system with known "discrete non-degenerate spectrum" e_n . As some physical appearances, a few known physical systems in the two mentioned categories have been considered. Finally, since the construction of nonclassical states is a central topic of quantum optics, nonclassical features and quantum statistical properties of the introduced states have been investigated by evaluating single- and two-mode squeezing, $su(1,1)$ -squeezing, Mandel parameter and antibunching effect (via g -correlation function) as well as some of their generalized forms we have introduced in the present paper.

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1 Introduction

Simultaneous with quantum mechanics birth, Schrödinger while working on quantum harmonic oscillator, introduced coherent states [1]. But it was further to prominent

investigations of Glauber and Sudarshan [2, 3] on genesis of laser as first accessible source of standard coherent states that studies on these states have been started, strictly. Many attempts in recent decades have been made along generalization of standard coherent states. Nevertheless, all of the generalized coherent states can be displayed in three main categories. Based on algebraic (q -deformed [4] and nonlinear coherent states [5, 6]), symmetric considerations (coherent states of $SU(1, 1)$ and $SU(2)$ group [7, 8]) and finally the coherent states associated to potentials other than harmonic oscillator [9, 10, 11]. However, in all of the above states the quanta involved are uncharged. Pair coherent states were introduced in [12]. The usefulness of these states for the description of pion production in high-energy collisions is also clarified. But later they have been called as "charge coherent states" in [13] and again as "pair coherent states" in [14, 15]. Any way, the notion of "charge coherent states" has proved it's successive usefulness in many areas of physics researches. As a few examples one may refer to elementary particles physics [16, 17], nuclear physics [18], quantum physics [19], quantum field theory [20, 21] and quantum optics [22, 23] fields. These states are generated through physical schemes in the quantum optics studies [22, 24, 25, 26]. The "standard charge coherent state" are simultaneously eigenstates of two commuting operators, i.e., *charge operator* given by

$$Q = a_1^\dagger a_1 - a_2^\dagger a_2, \quad (1)$$

and *pair annihilation operator* $a_1 a_2$ with eigenvalues q and ξ , respectively, i.e.,

$$\begin{aligned} Q|\xi, q\rangle &= q|\xi, q\rangle, \\ a_1 a_2 |\xi, q\rangle &= \xi|\xi, q\rangle. \end{aligned} \quad (2)$$

The explicit form of the eigenkets have been called as charge coherent states represent a well-known class of states within the general theory of coherent states and in the relevant literature [27] read as

$$\begin{aligned} |\xi, q\rangle^{(+)} &= \mathcal{N}_q^{(+)} (|\xi|^2)^{-1/2} \sum_{n=0}^{\infty} \frac{\xi^n}{\sqrt{n!(n+q)!}} |n+q, n\rangle, \quad q \geq 0, \\ |\xi, q\rangle^{(-)} &= \mathcal{N}_q^{(-)} (|\xi|^2)^{-1/2} \sum_{n=0}^{\infty} \frac{\xi^n}{\sqrt{n!(n-q)!}} |n, n-q\rangle, \quad q \leq 0, \end{aligned} \quad (3)$$

where the kets $|m, n\rangle$ are the two-mode number states and $\mathcal{N}_q^{(\pm)}$ are the normalization factors may be determined as

$$\mathcal{N}_q^{(\pm)} = |\xi|^{\pm q} I_q(\mp 2|\xi|) \quad (4)$$

with $I_q(x)$ as the first order of modified Bessel function of the first kind. At this point it is worth to mention that from the group theoretical point of view the above states are nothing but a $SU(1, 1)$ coherent states. This is due to the fact that the operators $a_1 a_2$ and

its conjugate are two of the three generators in the Schwinger-like definition of $SU(1, 1)$. In addition, the eigenvalues of the Q charge operator is essentially the representation index of the principle-series representation of $SU(1, 1)$ group where $C = (1 - Q^2)/4$ represents the corresponding Casimir operator. The states in (6) are simultaneously eigenstates of k_- and C .

Then, Liu constructed even and odd charge coherent states that are eigenstates of both operators Q and $(a_1 a_2)^2$ [28], i.e.,

$$\begin{aligned} Q |\xi, q\rangle_{e(o)}^{(\pm)} &= q |\xi, q\rangle_{e(o)}^{(\pm)}, \\ (a_1 a_2)^2 |\xi, q\rangle_{e(o)}^{(\pm)} &= \xi^2 |\xi, q\rangle_{e(o)}^{(\pm)}. \end{aligned} \quad (5)$$

These latter states are explicitly defined as

$$|\xi, q\rangle^{(\pm)} = \mathcal{N}'_q^{(\pm)} (|\xi, q\rangle \pm |-\xi, q\rangle) \quad (6)$$

where

$$\mathcal{N}'_q^{(\pm)} = \frac{1}{\sqrt{2}} \left[\pm q \pm (\mathcal{N}_q^{(\pm)})^2 |\xi|^{-q/2} J_q(\sqrt{2}|\xi|) \right] \quad (7)$$

in which the closed form of $\mathcal{N}_q^{(\pm)}$ has been defined in (4) and $J_q(x)$ is the Bessel function of the first kind. Liu *et al* followed k -component coherent states idea in [29] and constructed k -component charge coherent states which are eigenstates of "charge" and the k th-powers of "pair boson annihilation" operators [30]. The well-known \underline{q} -deformed coherent states [4] are defined as the right eigenstates of $b = f_{\underline{q}}(n)a$, with

$$f_{\underline{q}}(n) = \sqrt{\frac{\underline{q}^{n+1} - \underline{q}^{-n-1}}{(n+1)(\underline{q} - \underline{q}^{-1})}}, \quad (8)$$

provided that $0 < \underline{q} \leq 1$. It is worth mentioning that the specific notation \underline{q} in the description of $f_{\underline{q}}(n)$ is chosen here to distinguish it from the eigenvalues q of Q operator. The nonlinearity of q -deformed oscillators were realized in the dependence of frequency on the intensity ($n = a^\dagger a$) of light. Using this deformation Chatourvedi and Srinivasan constructed \underline{q} -deformed charge coherent states that are eigenstates of both charge operator in (1) and pair \underline{q} -boson annihilation operators [31], $b_1 b_2$ with $b_i = f_{\underline{q}}(n_i) a_i$ for $i = 1, 2$. The even and odd \underline{q} -deformed charge coherent states are eigenstates of the square of the pair \underline{q} -deformed annihilation operators $(b_1 b_2)^2$ and the charge operator [32]. Liu by using the concept of k -component \underline{q} -deformed coherent states [33] introduced the k -component \underline{q} -deformed charge coherent states [34] that are eigenstates of charge operator in (1) and k th-power of \underline{q} -deformed boson operator $(b_1 b_2)^k$.

On the other side, nonlinear coherent states were first introduced by de Matoes Filho and Vogel [5] and Man'ko *et al* [6]. These states are defined as right eigenstates of the f -deformed annihilation operators $A = af(n)$, where $f(n)$ is the nonlinearity function.

As established by Vogel *et al* [5, 35] a special set of these quantum states may be generated as the stationary states of the center-of-mass motion of a trapped ion far from the Lamb-Dicke regime. There has been shown that there are so many generalized coherent states that can be put in this important category with some special nonlinearity functions [36, 37]. Indeed, these states provide a powerful method which can unify the mathematical structure of a lot of generalized coherent states introduced in the literature. So, a reasonable and natural extension of the standard and q -deformed charge states is provided by the notion of "*f-deformed charge coherent states*". The main purpose of the present work is to introduce a general formalism for the construction of a wide classes of charge coherent states based on the nonlinear coherent states method. It is worth to mention that upon the relation between nonlinearity function of nonlinear coherent states and the Hamiltonian of the quantum systems proposed by one of us [36, 38] the formalism can be extended to solvable quantum systems with discrete non-degenerate spectra. We hope that the introduced nonlinear charge coherent states in the present paper which stream to nonlinear physical systems may find their useful applications in various fields of researches, as the standard charge coherent states [16]-[26].

The paper organized as follows. In section 2 we outline the general structure of nonlinear charge coherent states. The nonlinear algebraic realization of the $su_Q(1, 1)$ generators for the introduced states is established in section 3. Then, by symmetric and antisymmetric superposition of nonlinear charge coherent states, even and odd nonlinear charge coherent states have been constructed in section 4. Next, in section 5 we study the nonclassical properties and quantum statistics of the introduced states via investigating single- and two-mode squeezing in addition to $su(1, 1)$ -squeezing, Mandel parameter, and two-mode second order correlation function together with all of their generalized forms will be introduced in the present work. Finally, in section 6 we apply our presented formalism to a few physical situations either solvable quantum systems with known "discrete spectrum" e_n or nonlinear oscillator systems with known "nonlinearity function" $f(n)$. For instance, "Sudarshan harmonious states" and " $SU(1, 1)$ coherent states" as the first type and "hydrogen-like spectrum", "Pöschl-Teller" and "infinite well potentials" as the second type will be considered and discussed.

2 Introducing the nonlinear charge coherent states

In this section using the nonlinear coherent states method [6, 36] we introduce the nonlinear charge coherent states in a general structure. Consider f -deformed ladder operators

$$A_i = a_i f(n_i), \quad A_i^\dagger = f^\dagger(n_i) a_i^\dagger, \quad (9)$$

where a_i , a_i^\dagger and $n_i = a_i^\dagger a_i$, $i = 1, 2$ are respectively boson annihilation, creation and number operators of i th mode, and $f(n)$ is a nonlinear function characterizes the physical

systems. The pair f -deformed boson annihilation operator $A_1 A_2$ and charge operator commute with together, i.e., $[Q, A_1 A_2] = 0$, where charge operator keeps its previous definition in (1). Thus, the latter two operators should satisfy eigenvalue equations as follow

$$\begin{aligned} Q|\xi, q, f\rangle &= q|\xi, q, f\rangle, \\ A_1 A_2|\xi, q, f\rangle &= \xi|\xi, q, f\rangle, \end{aligned} \quad (10)$$

where $\xi \in \mathcal{C}$ and q is an integer has been called charge number. These eigenstates have generally the following expansion

$$|\xi, q, f\rangle = \sum_{n,m=0}^{\infty} c_{n,m} |n, m\rangle. \quad (11)$$

Substituting (11) in (10) and calculating expansion coefficients one straightforwardly obtains two distinct states as follows

$$|\xi, q, f\rangle^{(+)} = \mathcal{N}^{(+)} (|\xi|^2)^{-1/2} \sum_{n=0}^{\infty} \xi^n \frac{1}{\sqrt{n! [n+q]! [f(n)]! [f(n+q)]!}} |n+q, n\rangle, \quad q \geq 0, \quad (12)$$

$$|\xi, q, f\rangle^{(-)} = \mathcal{N}^{(-)} (|\xi|^2)^{-1/2} \sum_{n=0}^{\infty} \xi^n \frac{\xi^n}{\sqrt{n! [n-q]! [f(n)]! [f(n-q)]!}} |n, n-q\rangle, \quad q \leq 0. \quad (13)$$

The latter states can be put in the following single expression

$$\begin{aligned} |\xi, q, f\rangle &= \mathcal{N} (|\xi|^2)^{-1/2} \sum_{n=0}^{\infty} \frac{\xi^n}{\sqrt{n! [n+|q|]! [f(n)]! [f(n+|q|)]!}} \\ &\times \left| n + \frac{q+|q|}{2}, n - \frac{q-|q|}{2} \right\rangle, \end{aligned} \quad (14)$$

with normalization constant given by

$$\mathcal{N} (|\xi|^2) = \sum_{n=0}^{\infty} \frac{|\xi|^{2n}}{n! [n+|q|]! \{ [f(n)]! [f(n+|q|)]! \}^2}. \quad (15)$$

Note that in obtaining (12)-(15) using the conventional definitions

$$[f(n)]! \doteq f(n)f(n-1)\dots f(1), \quad [f(0)]! \doteq 1, \quad (16)$$

we have defined

$$\begin{aligned} [n+|q|]! &\doteq (n+|q|)(n-1+|q|)\dots(1+|q|), & [|q|]! &\doteq 1, \\ [f(n+|q|)]! &\doteq f(n+|q|)f(n-1+|q|)\dots f(1+|q|), & [f(|q|)]! &\doteq 1. \end{aligned} \quad (17)$$

To endorse our formalism we can substitute $f(n) = 1$ in above relations and check that standard charge coherent state in (6) will be obtained. It is worth noticing that by substituting the \underline{q} -deformation function (8) in the above results one can not expect the exact result of \underline{q} -deformed charge coherent states in [32]. This is due to the fact that we began our presented formalism of nonlinear charge coherent states with the deformed annihilation operator $A = af(n)$, while in [32] the authors used the definition $A = f_{\underline{q}}(n)a$. Moreover, due to the relation $f(n)a = af(n-1)$ replacing n with $n-1$, the consistency of the formalism will be revealed. We would like to mention that the q -deformation nonlinearity in Eq. (8) has no zeroes at positive integers, and this feature is not changed by replacing n with $n-1$. So in both cases the Ston-Von Numman theorem can be extended to the case of the related operators A and A^\dagger . Also the ladder operators are irreducible over the Fock space.

3 Deformed algebraic realization of the introduced states

Liu considered the standard (undeformed) $su(1,1)$ Lie algebra generators in terms of the two-mode boson operators as [28]

$$k_- = a_1 a_2, \quad k_+ = a_1^\dagger a_2^\dagger, \quad k_0 = \frac{1}{2}(a_1^\dagger a_1 + a_2^\dagger a_2 + 1). \quad (18)$$

The relevant coherent states are well-known charge coherent states or Barut-Girardello coherent states [7, 12, 13]. Keeping in mind the Liu work and following the path of Hu and Chen in [39], we choose the associated generators for our nonlinear charge coherent states using the f -deformed ladder operators in (9) as follows:

$$K_- = A_1 A_2, \quad K_+ = A_1^\dagger A_2^\dagger, \quad [K_-, K_+] = 2K_0, \quad (19)$$

where K_0 can be calculated as

$$K_0 = \frac{1}{2}[(n_1 + 1)f^2(n_1 + 1)(n_2 + 1)f^2(n_2 + 1) - n_1 f^2(n_1)n_2 f^2(n_2)]. \quad (20)$$

Obviously one has $K_- = (K_+)^{\dagger}$, $(K_-)^{\dagger} = K_+$ and $K_0 = K_0^{\dagger}$. Now it can be easily checked that the following commutation relations hold:

$$[K_0, K_-] = -K_- g(n_1, n_2), \quad [K_0, K_+] = g(n_1, n_2)K_+, \quad (21)$$

where we have defined:

$$\begin{aligned} g(n_1, n_2) &= \frac{1}{2}[(n_1 + 1)f^2(n_1 + 1)(n_2 + 1)f^2(n_2 + 1) - n_1 f^2(n_1) \\ &\times n_2 f^2(n_2) + (n_1 - 1)f^2(n_1 - 1)(n_2 - 1)f^2(n_2 - 1)]. \end{aligned} \quad (22)$$

Thus, we established the nonlinear deformed algebra denoted by us as $su_Q(1,1)$ for the nonlinear charge coherent states in (14).

4 Even and odd nonlinear charge coherent states

We can construct the even and odd nonlinear charge coherent state via symmetric and antisymmetric superposition of nonlinear charge coherent states introduced in (12) and (13) such as:

$$\begin{aligned} |\xi, q, f\rangle_{e(o)}^{(\pm)} &= \frac{1}{2} \mathcal{N}^{(\pm)}(|\xi|^2)^{1/2} \mathcal{N}_{e(o)}^{(\pm)}(|\xi|^2)^{-1/2} \\ &\times (|\xi, q, f\rangle^{(\pm)} \pm |-\xi, q, f\rangle^{(\pm)}), \quad q \geq 0. \end{aligned} \quad (23)$$

The explicit form of the above states for the case $q \geq 0$ then straightforwardly may be obtained separately as

$$\begin{aligned} |\xi, q, f\rangle_e^{(+)} &= \mathcal{N}_e^{(+)}(|\xi|^2)^{-1/2} \sum_{n=0}^{\infty} \xi^{2n} \\ &\times \frac{1}{\sqrt{(2n)! [2n+q]! [f(2n)]! [f(2n+q)]!}} |2n+q, 2n\rangle, \end{aligned} \quad (24)$$

$$\begin{aligned} |\xi, q, f\rangle_o^{(+)} &= \mathcal{N}_o^{(+)}(|\xi|^2)^{-1/2} \sum_{n=0}^{\infty} \frac{\xi^{2n+1}}{\sqrt{(2n+1)! [2n+1+q]!}} \\ &\times \frac{1}{[f(2n+1)]! [f(2n+1+q)]!} |2n+1+q, 2n+1\rangle. \end{aligned} \quad (25)$$

The two normalization constants can be calculated as follows:

$$\mathcal{N}_{e(o)}^{(+)}(|\xi|^2) = \frac{1}{2} \mathcal{N}^{(+)}(|\xi|^2) \pm \frac{1}{2} \sum_{n=0}^{\infty} \frac{(-|\xi|)^n}{n! [n+q]! \{[f(n)]! [f(n+q)]!\}^2}, \quad (26)$$

where $\mathcal{N}^{(+)}(|\xi|^2)$ may be obtained from (15) when $q \geq 0$. By replacing $2n$ with $2n-q$ in relations (24) and (25) one can obtain the associated states for $q \leq 0$. It is easy to check that these states may also be obtained as the eigenstates of squared of pair f -deformed annihilation operators with eigenvalues ξ^2 , i.e.,

$$(A_1 A_2)^2 |\xi, q, f\rangle_{e(o)} = \xi^2 |\xi, q, f\rangle_{e(o)}. \quad (27)$$

Also, the following relations hold

$${}_{e(o)}\langle \xi, q, f | \xi, q', f' \rangle_{e(o)} = \mathcal{N}_{e(o)}(|\xi|^2)^{-1/2} \mathcal{N}_{e(o)}(|\xi'|^2)^{-1/2} \mathcal{N}_{e(o)}(|\xi^* \xi'|) \delta_{q, q'}, \quad (28)$$

$${}_e\langle \xi, q, f | \xi', q, f \rangle_o = 0. \quad (29)$$

The Dirac delta function in (28) explains that the even nonlinear charge coherent states are orthogonal to each other relative to charge number. Similar situation holds for odd nonlinear charge coherent states, too. In addition, the relation (29) indicates that even and odd nonlinear charge coherent state with identical charge number and clearly the same nonlinearity function are orthogonal.

5 Nonclassicality of the introduced states

Motivations to introduce generalized coherent states theoretically and to produce them in the laboratory are mainly due to their nonclassical properties which their usefulness in sensitive measurements is a well-known subject. In this section we would like to illustrate what nonclassical properties will be considered by us to investigate the nonclassicality features of the introduced states. To achieve this purpose we will check various properties such as single- and two mode-squeezing, $su(1, 1)$ -squeezing, Mandel parameter and second order correlation function (also their new generalized forms which we will define). It is worth mentioning that only one of the latter properties is sufficient for a states to be considered as nonclassical states.

5.1 Squeezing effects

Different squeezing parameters will be outlined in this subsection.

5.1.1 Single-mode squeezing and it's generalization

Usually quadrature operators of filed in two-mode are defined as follows

$$y_1 = \frac{a_1^\dagger + a_1}{2}, \quad y_2 = \frac{i(a_1^\dagger - a_1)}{2}, \quad (30)$$

$$z_1 = \frac{a_2^\dagger + a_2}{2}, \quad z_2 = \frac{i(a_2^\dagger - a_2)}{2}, \quad (31)$$

where y_i , $i = 1, 2$, indicates to the quadratures of the first mode and z_i to the second mode. They satisfy the canonical commutation relations:

$$[y_1, y_2] = \frac{i}{2}, \quad [z_1, z_2] = \frac{i}{2}. \quad (32)$$

So, the uncertainty relation for above conjugate operators read respectively as

$$\langle (\Delta y_1)^2 \rangle \langle (\Delta y_2)^2 \rangle \geq \frac{1}{16}, \quad \langle (\Delta z_1)^2 \rangle \langle (\Delta z_2)^2 \rangle \geq \frac{1}{16}, \quad (33)$$

where in these relations and all other cases which will be followed the uncertainties in any partner of a set of conjugate operator defined by $\langle (\Delta x)^2 \rangle \doteq \langle x^2 \rangle - \langle x \rangle^2$. Therefore, single-mode squeezing occurs in the first mode for any quantum state if

$$\langle (\Delta y_i)^2 \rangle < \frac{1}{4}, \quad i = 1 \text{ or } 2, \quad (34)$$

and similarly for the second mode. The corresponding fluctuations with respect to the states introduced in (14) are give by

$$\begin{aligned} \langle (\Delta y_1)^2 \rangle &= \langle (\Delta y_2)^2 \rangle = \frac{1}{4} \left\{ 2\mathcal{N}(|\xi|^2)^{-1} \right. \\ &\times \left. \sum_{n=0}^{\infty} \frac{|\xi|^{2n}}{(n-1)! [n+|q|]! \{ [f(n)]! [f(n+|q|)]! \}^2} + 1 \right\}, \end{aligned} \quad (35)$$

and

$$\begin{aligned} \langle (\Delta z_1)^2 \rangle &= \langle (\Delta z_2)^2 \rangle = \frac{1}{4} \left\{ 2\mathcal{N}(|\xi|^2)^{-1} \right. \\ &\times \left. \sum_{n=0}^{\infty} \frac{|\xi|^{2n}}{n! [n-1+|q|]! \{ [f(n)]! [f(n+|q|)]! \}^2} + 1 \right\}. \end{aligned} \quad (36)$$

Equations (35) and (36) show that the inequalities $\langle (\Delta y_i)^2 \rangle \geq \frac{1}{4}$ and $\langle (\Delta z_i)^2 \rangle \geq \frac{1}{4}$ always hold for $i = 1$ or 2 . These results indicate that single-mode squeezing can not occur for any arbitrary function $f(n)$.

Changing the boson operators in relations (30) and (31) by f -deformed ladder operator, we define the "generalized quadrature operators" respectively for first and second mode as follows

$$Y_1 = \frac{A_1^\dagger + A_1}{2}, \quad Y_2 = \frac{i(A_1^\dagger - A_1)}{2}, \quad (37)$$

$$Z_1 = \frac{A_2^\dagger + A_2}{2}, \quad Z_2 = \frac{i(A_2^\dagger - A_2)}{2}. \quad (38)$$

These two pairs of quadratures satisfy the noncanonical commutation relations:

$$[Y_1, Y_2] = \frac{i}{2} [(n_1 + 1)f^2(n_1 + 1) - n_1 f^2(n_1)], \quad (39)$$

$$[Z_1, Z_2] = \frac{i}{2} [(n_2 + 1)f^2(n_2 + 1) - n_2 f^2(n_2)]. \quad (40)$$

So, the uncertainty relations are respectively given by

$$\langle (\Delta Y_1)^2 \rangle \langle (\Delta Y_2)^2 \rangle \geq \frac{1}{16} \left| \langle (n_1 + 1)f^2(n_1 + 1) - n_1 f^2(n_1) \rangle \right|^2, \quad (41)$$

$$\langle (\Delta Z_1)^2 \rangle \langle (\Delta Z_2)^2 \rangle \geq \frac{1}{16} \left| \langle (n_2 + 1)f^2(n_2 + 1) - n_2 f^2(n_2) \rangle \right|^2. \quad (42)$$

A state is said to be single-mode squeezed in Y_i if

$$\langle (\Delta Y_i)^2 \rangle < \frac{1}{4} \left| \langle (n_i + 1)f^2(n_i + 1) - n_i f^2(n_i) \rangle \right|^2, \quad (43)$$

and in Z_i if

$$\langle (\Delta Z_i)^2 \rangle < \frac{1}{4} \left| \langle (n_i + 1)f^2(n_i + 1) - n_i f^2(n_i) \rangle \right|^2. \quad (44)$$

where in (43) and (44) $i = 1$ or 2 . It can be easily checked that single-mode squeezing can not hold for arbitrary $f(n)$ function similar to (35) and (36).

5.1.2 Two-mode squeezing and it's generalization

Following the definitions in [28, 40] with the help of (30) and (31) we now introduce the two-mode hermitian quadrature operators:

$$w_1 = \frac{y_1 + z_1}{\sqrt{2}}, \quad w_2 = \frac{y_2 + z_2}{\sqrt{2}}, \quad (45)$$

that satisfy the canonical commutation relation

$$[w_1, w_2] = \frac{i}{2}. \quad (46)$$

The uncertainty relation is simply obtained as

$$\langle (\Delta w_1)^2 \rangle \langle (\Delta w_2)^2 \rangle \geq \frac{1}{16}. \quad (47)$$

The two-mode squeezing for nonlinear charge coherent states may be observed if

$$\langle (\Delta w_i)^2 \rangle < \frac{1}{4}, \quad i = 1 \text{ or } 2. \quad (48)$$

The fluctuations in w_1 and w_2 may be calculated straightforwardly, where again it is observed that there are not two-mode squeezing with respect to introduced states for any arbitrary $f(n)$ function.

Generalizing the proposal, by replacing a and a^\dagger respectively with A and A^\dagger in relations (45) we can define new "generalized two-mode quadrature operators" as

$$W_1 = \frac{Y_1 + Z_1}{\sqrt{2}}, \quad W_2 = \frac{Y_2 + Z_2}{\sqrt{2}}, \quad (49)$$

where Y_i and Z_i ($i = 1, 2$) defined in (37) and (38). These quadratures satisfy the following commutation relation

$$\begin{aligned} [W_1, W_2] &= \frac{i}{4} [(n_1 + 1)f^2(n_1 + 1) - n_1 f^2(n_1) \\ &+ (n_2 + 1)f^2(n_2 + 1) - n_2 f^2(n_2)]. \end{aligned} \quad (50)$$

Uncertainty relation corresponding to these conjugate quadratures read as

$$\langle (\Delta W_1)^2 \rangle \langle (\Delta W_2)^2 \rangle \geq \frac{1}{4} |\langle [W_1, W_2] \rangle|^2. \quad (51)$$

So, generalized two-mode squeezing for nonlinear charge coherent states may be observed if

$$S_{W_i} = \langle (\Delta W_i)^2 \rangle - \frac{1}{2} |\langle [W_1, W_2] \rangle| < 0, \quad (52)$$

where $i = 1$ or 2 .

5.1.3 $su(1, 1)$ -squeezing and it's generalization

Following the Wodkiewicz and Eberly proposal in [41] we now consider two hermitian quadratures that constructed from usual $su(1, 1)$ generators k_+ , k_- were defined in (18) as

$$x_1 = \frac{k_+ + k_-}{2}, \quad x_2 = \frac{i(k_+ - k_-)}{2}, \quad (53)$$

satisfy the commutation relation

$$[x_1, x_2] = \frac{i}{2}(n_1 + n_2 + 1). \quad (54)$$

The uncertainty relation for these quadratures will be written as follows

$$\langle (\Delta x_1)^2 \rangle \langle (\Delta x_2)^2 \rangle \geq \frac{1}{4} |\langle [x_1, x_2] \rangle|^2. \quad (55)$$

A state is said to exhibit $su(1, 1)$ -squeezing if

$$S_{x_i} = \langle (\Delta x_i)^2 \rangle - \frac{1}{2} |\langle [x_1, x_2] \rangle| < 0, \quad i = 1 \text{ or } 2. \quad (56)$$

Following the Liu proposal [32] we now consider two hermitian f -deformed quadratures as

$$X_1 = \frac{K_+ + K_-}{2}, \quad X_2 = \frac{i(K_+ - K_-)}{2}, \quad (57)$$

where K_+ and K_- were defined in (19). These quadratures satisfy the commutation relation

$$[X_1, X_2] = \frac{i}{2} [(n_1 + 1)f^2(n_1 + 1)(n_2 + 1)f^2(n_2 + 1) - n_1 f^2(n_1) n_2 f^2(n_2)]. \quad (58)$$

The uncertainty relation for these quadratures will be written as follows

$$\langle (\Delta X_1)^2 \rangle \langle (\Delta X_2)^2 \rangle \geq \frac{1}{4} |\langle [X_1, X_2] \rangle|^2. \quad (59)$$

A state may exhibit generalized $su(1, 1)$ -squeezing if

$$S_{X_i} = \langle (\Delta X_i)^2 \rangle - \frac{1}{2} |\langle [X_1, X_2] \rangle| < 0, \quad i = 1 \text{ or } 2. \quad (60)$$

5.2 Quantum statistics of nonlinear charge coherent states

A familiar quantity in quantum statistics is the photon count probability. The probability of being n photons in first mode and $n - q$ photons in the second mode for nonlinear charge coherent states in (14) may be given in the following closed form expression

$$P\left(n + \frac{q + |q|}{2}, n - \frac{q - |q|}{2}\right) = \mathcal{N}(|\xi|^2)^{-1} \frac{|\xi|^{2n}}{n! [n + |q|]!} \frac{1}{\{[f(n)]! [f(n + |q|)]!\}^2}. \quad (61)$$

Moreover, there are two measures that may be used to study the fluctuations of quanta number distribution as the "Mandel's Q parameter" and "second order correlation function" which will be considered by us.

5.2.1 Mandel parameter and it's generalization

Mandel parameter ordinarily is defined as follows [42]:

$$Q_i^a = \frac{\langle n_i^2 \rangle - \langle n_i \rangle^2}{\langle n_i \rangle} - 1, \quad (62)$$

where the superscript "a" indicates that we have used bosonic ladder operators and hence $n_i = a_i^\dagger a_i$, and the subscript $i = 1, 2$ indicates the first and second mode, respectively.

At this point we would like to generalize the definition (62) to the following form

$$Q_i^A = \frac{\langle N_i^2 \rangle - \langle N_i \rangle^2}{\langle N_i \rangle} - 1, \quad (63)$$

where again the superscript "A" indicates that we have used $N_i = A_i^\dagger A_i$, and the subscript $i = 1, 2$ indicates the first and second mode, respectively.

The negativity of Mandel parameter in (62) or its generalized form in (63) indicates that the quantum statistics is sub-Poissonian, which shows the nonclassicality of the state.

5.2.2 Two-mode correlation function and it's generalization

Generalizing correlation function definition of a single-mode field, Liu defined the two-mode correlation function as [28]:

$$g = \frac{\langle (n_1 n_2)^2 \rangle}{\langle n_1 n_2 \rangle^2}. \quad (64)$$

Two-photon number distribution is related to this measure, where it determines the two-photon correlations degree in a two-mode field. If $g < 1$ the state is said to have two-mode antibunching characterizes the nonclassically of the state.

Following Liu definition in [28] we now define the "*generalized two-mode correlation function*" as:

$$G = \frac{\langle (N_1 N_2)^2 \rangle}{\langle N_1 N_2 \rangle^2}, \quad (65)$$

where $N_i = A_i^\dagger A_i$ for $i = 1, 2$. If $G < 1$ the state is said to have generalized two-mode antibunching, i.e., it possesses nonclassicality behavior.

5.3 Nonclassicality of even and odd nonlinear charge coherent states

We followed this section with a few words on the nonclassicality of the even and odd nonlinear charge coherent states. It can be straightforwardly checked that the single- and two-mode squeezing (and their generalized forms) may not be occurred for even and

odd nonlinear charge coherent states. This observations are the same as the previously mentioned results for the original nonlinear charge coherent states. But, other criteria such as $su(1,1)$ -squeezing, Mandel parameter and the generalized form of it introduced respectively in (62) and (63), also the two-mode correlation function as well as it's generalized form respectively introduced in (64) and (65), will be discussed for even (and odd) nonlinear charge coherent state.

The probability distributions for even and odd nonlinear charge coherent states are respectively given by

$$P_e(2n, 2n + |q|) = \mathcal{N}_e(|\xi|^2)^{-1} \frac{|\xi|^{2n}}{(2n)! [2n + |q|]! \{[f(2n)]! [f(2n - q)]!\}^2}, \quad (66)$$

and

$$\begin{aligned} P_o(2n + 1, 2n + 1 + |q|) &= \mathcal{N}_o(|\xi|^2)^{-1} \{(2n + 1)! [2n + 1 + |q|]!\}^{-1} \\ &\times \frac{|\xi|^{2n+1}}{\{[f(2n + 1)]! [f(2n + 1 + |q|)]!\}^2}. \end{aligned} \quad (67)$$

It is clear that $P_e(2n + 1, 2n + 1 + |q|) = 0$ and $P_o(2n, 2n + |q|) = 0$. Oscillatory nature of these distributions indicates the obvious nonclassically feature of the latter states.

6 New classes of nonlinear charge coherent states corresponding to physical systems and their non-classical features

In order to illustrate the physical applications of the presented formalism in the paper, let us apply the structure on some physical systems which the associated "*nonlinear coherent states*" and so their corresponding nonlinearity functions, or their "*discrete spectrum*" have already been known. Recalling the normal-ordered Hamiltonian of the nonlinear oscillator it will be enough for our intention to introduce the discrete spectrum e_n or the nonlinearity function $f(n)$. According to the proposed method by one of us in [36, 37] these two physical quantities are simply related by $e_n = n f^2(n)$. Thus, knowing the explicit form of e_n one can obtain the corresponding nonlinearity function as $f(n) = \sqrt{\frac{e_n}{n}}$ and vice versa. Then, substituting a special nonlinearity function $f(n)$ into "equations (12) and (13)" or "equations (24) and (25)" give readily the explicit form of "nonlinear charge coherent states" or "even and odd nonlinear charge coherent states" associated to each particular system, respectively. Therefore, to economize in space the explicit form of nonlinear charge coherent states associated to particular physical systems will be introduced in the continuation of the paper have not been given.

It must be noticed that since we are interested in the nonclassical properties of the states of the field, and due to the exitance of numerous nonclassical criteria have been

outlined in the paper, we will mainly focus on the presentation of the numerical results which the nonclassicality aspects of the introduced states may be deeply clarified.

6.1 Application to physical systems with known discrete spectra

In this subsection we will deal with some classes of nonlinear coherent states associated to quantum systems whose corresponding spectra are known. Recall that it contains only a few quantum mechanical systems in physical context. Among them we will pay attention to Pöschl-Teller potential and Hydrogen-like spectrum.

Example 1, Pöschl-Teller potential: The interest in this potential and its coherent states is due to various applications in many fields of physics particularly in atomic and molecular physics. The associated Gazeau-Klauder coherent states have been demonstrated and discussed nicely by Antonie *et al* [43] with the convergence radius $R = \infty$. The corresponding spectrum is $e_n = n(n + \nu)$, so the associated nonlinearity function is [36]

$$f_{PT}(n) = \sqrt{n + \nu}, \quad \nu \geq 2, \quad (68)$$

where the case $\nu = 2$ corresponds to infinite potential well. We fixed the parameters $q = 2$ and $\nu = 3$ in all computational calculations which take into account this particular system. Our numerical results are as follows. From figure 1 it is seen that $su(1, 1)$ -squeezing in x_1 quadrature occurs for nonlinear charge coherent states (12) corresponding to the latter potential. While for the standard charge states no such squeezing is seen in neither of the quadratures. Our further numerical calculations show that the $su(1, 1)$ -squeezing for the corresponding nonlinear charge coherent state occurs in X_1 quadrature, too. Figure 2 shows that for both classes of even charge states, either standard or nonlinear, $su(1, 1)$ -squeezing has been occurred in x_2 quadrature in some regions of $x = |\xi|^2$. Figure 3 shows that the same results may be obtained qualitatively for X_1, X_2 . But in this case the strength of squeezing in X_2 for the nonlinear charge states is much higher than the standard one. According to our numerical results concerning the correlation g -factor for latter potential and all of the introduced nonlinear charge states is less than 1. For instance, see figure 4 which indicates the two-mode antibunching effect for the nonlinear charge coherent states as well as the same quantity for the standard charge states. It is clear from the figure that while $g < 1$ nearly in all range of x , for the standard charge states it is approximated to 1, i.e. the same as canonical (classical) coherent states. Figure 5 shows the generalized correlation G -factor for odd nonlinear charge coherent state together with the standard charge states, from which we observe that $G < 1$ for nonlinear charge states in a wide region of x , indicates the nonclassicality of the nonlinear odd charge states. This quantity for the standard charge states also becomes negative, but only in a finite region of space, i.e., $x < 1.75$. Mandel parameter in (62) and its generalized form in (63) for the nonlinear charge states in both modes can be negative. For instance, as it is shown in figure 6 Mandel parameter has been plotted using (62) for

first mode, which is negative for arbitrary x , either standard or nonlinear charge states. But it is noticeable that in contrast to all previous cases in which the nonclassicality of nonlinear charge states become more high light with respect to the standard one, concerning with the latter quantity although both are negative, the negativity of the nonlinear charge states are less than standard one.

Example 2, Hydrogen-like spectrum: We now choose the hydrogen-like spectrum whose the corresponding coherent states have been a long standing subject and discussed frequently in the literature [44]. The one-dimensional model of such a system with the Hamiltonian $\hat{H} = -\frac{\omega}{(n+1)^2}$ and the eigenvalues $E_n = -\omega/(n+1)^2$ has been considered ($\omega = me^4/2$, and $n = 0, 1, 2, \dots$). But to be consistent with the proposed formalism in [37] one must take the shifted Hamiltonian $e_n = 1 - \frac{1}{(n+1)^2}$ where we have taken $\omega \equiv 1$. Thus, for the nonlinearity function corresponding to such a system one obtains [37]

$$f_H(n) = \frac{\sqrt{n+2}}{n+1}. \quad (69)$$

The nonlinear coherent states corresponding to this nonlinearity function may be defined on the unit disk centered at the origin. We fixed the charge parameter $q = 2$ in all cases which take into account the hydrogen spectrum. Our numerical calculations show that the $su(1,1)$ -squeezing occurs in x_2 quadrature when the nonlinear charge coherent states and even nonlinear charge coherent states take into account (see for example figure 7 which is plotted for nonlinear charge coherent states). It is also seen from figure 7 that for the standard charge states the graphs of squeezing coincide with the horizontal axis. So nonlinearity causes the nonclassicality considering this criteria. According to our computational results, for nonlinear charge coherent states associated to hydrogen-like atoms the Heisenberg uncertainty relation (59) is saturated. Therefore, the obtained states can be regarded as the *generalized intelligent states* [45] with respect to those quadratures. This situation also holds for standard charge states as it is seen from figure 1. For even nonlinear charge coherent states $su(1,1)$ -squeezing will be observed in X_1 quadratures, and the same behavior occurs for the standard charge states (see figure 8). For the associated odd nonlinear charge coherent states the inequality $g < 1$ holds in a finite region of x near the origin (see figure 9). As it is observed this quantity is approximated to zero for the standard charge states (so possess no nonclassicality criteria). Our numerical calculations shows that $G < 1$ for all of the permitted values of x associated to odd nonlinear charge coherent states. Based on our numerical results Mandel parameter Q_1^a for the first mode of the odd nonlinear charge coherent states is negative in some regions of x . The parameter Q_2^a for second mode with respect to all of the corresponding classes of charge coherent states will be negative in some regions of x . The generalized Mandel parameter Q_1^A will be negative for the first mode of even nonlinear charge coherent states while this quantity for the standard charge states is approximated to 1 (no classical feature) (see figure 10), and for odd one we obtained $Q_1^A \simeq -1$. For

second mode we obtained $Q_2^A \simeq -1$ for all introduced classes of corresponding charge states. Recall that Mandel parameter of number states as the most nonclassical states is also -1 .

To this end, the oscillatory nature of photon distributions for even and odd nonlinear charge coherent states associated to both above physical systems is an intrinsic feature of these states, so need not to be discussed.

6.2 Application to physical systems with known nonlinearity functions

In this subsection we will concern with some classes of nonlinear coherent states whose corresponding nonlinearity functions previously introduced in the literature. Particularly they are harmonious states and $SU(1, 1)$ coherent states.

Example 1, Harmonious states: Sudarshan introduced coherent states for simple unweighted shift operators that has named harmonious states with the nonlinearity function [46]

$$f_{HS}(n) = \frac{1}{\sqrt{n}}. \quad (70)$$

Our numerical outputs for nonlinear charge coherent states associated to this system are very closely the same as the results obtained for hydrogen atom. So, paying attention to its dual family [47, 48] with

$$f_{DHS}(n) = \sqrt{n}, \quad (71)$$

is preferred. This nonlinearity function appears in a natural way in Hamiltonians describing interaction with intensity-dependent coupling between a two-level atom and an electromagnetic field [49]. We again observe that the results of our calculations are very closely the same as those of Pöshl-Teller potential. The latter two facts may be expected from comparing the nonlinearity function (70) with (69) and (71) with (68).

Example 2, $SU(1, 1)$ states: Two distinct sets of generalized coherent states have been introduced in the literature associated to $su(1, 1)$ Lie algebra known as Barut-Girardello [50] and Gilmore-Perelomov [8, 51] coherent states. In [36] it has been demonstrated that Barut-Girardello coherent states can be created via the nonlinear coherent states method by means of nonlinearity function as

$$f_{BG}(n) = \sqrt{n + 2\kappa - 1}, \quad \kappa = 1/2, 1, 3/2, \dots. \quad (72)$$

The convergence radius of corresponding states is $R = \infty$. Our numerical results for the corresponding nonlinear charge coherent states and their even and odd counterparts indicate $su(1, 1)$ -squeezing with respect to nonlinear charge coherent states and even counterpart in x_2 quadrature. On the other side, for odd nonlinear charge coherent states $su(1, 1)$ -squeezing is observed in X_2 quadrature. For all of the corresponding charge

states g -factor is less than 1 showing the nonclassicality of states, for instance see figure 11 which has been plotted using the corresponding nonlinear charge coherent states. But as it is observed this quantity is also nearly 1 for the standard charge states. According to our numerical results G -factor for associated nonlinear charge coherent states is $\simeq 1$, while for even and odd ones it takes values less than 1. For second mode of $SU(1,1)$ nonlinear charge coherent states, generalized Mandel parameter for all of the charge states and modes will be negative at least in some finite regions (see figure 12 as an example). In this last case Q_A becomes negative for both the standard and nonlinear charge states.

Gilmore-Perelomov coherent states are defined by the nonlinearity function

$$f_{GP}(n) = 1/\sqrt{n + 2\kappa - 1}, \quad \kappa = 1/2, 1, 3/2, \dots \quad (73)$$

The convergence radius of corresponding states is the unit disk. The two sets of states Barut-Girardello and Gilmore-Perelomov are known as the dual pair [36]. Our numerical calculations show that the corresponding graphical results are closely the same as the charge states of hydrogen-like spectrum and harmonious states, qualitatively. This may be expected due to the functional form of the nonlinearity functions. The oscillatory nature of photon distribution for all of the associated even and odd nonlinear charge coherent states is again a clear fact.

7 Summary and conclusion

We sum up our presented results as follows. Based on the well-known nonlinear coherent states approach in quantum optics field, we proposed a formalism to introduce the nonlinear charge coherent states in a general framework associated to any classes of "nonlinear coherent states" as well as arbitrary "solvable quantum system" with known discrete non-degenerate spectra. The nonlinear deformed algebra of the nonlinear charge coherent states is realized as $su_Q(1,1)$. After deducing the explicit form of the proposed states, we constructed the even and odd nonlinear charge coherent states, through symmetric and antisymmetric superposition of the states, respectively. Various nonclassical properties of all classes of the introduced states associated to a few quantum physical systems have been investigated, numerically. The dependence of the nonclassicality nature on the "nonlinearity function" or "energy spectrum" has been clearly shown. According to our numerical results, generally, but not always the nonclassicality will be stronger when one deals with nonlinear charge coherent states comparing with the standard charge coherent states. It is noticeable that this conclusion depends on the choice of $f(n)$ and also the specific nonclassicality criteria. Besides the nonclassical properties which is the main interest in this field of research, a specific aspect of some of the introduced states are noticeable. We may refer to the nonlinear charge coherent state corresponding to hydrogen-like spectrum which behaves as intelligent state in X_1 and X_2 quadratures. To

this end, our formalism provides a general and simple scheme for the construction of a vast classes of charge coherent states and their even and odd counterparts associated to any particular nonlinearity function as well as arbitrary solvable quantum system with discrete spectra. For instance, the application of the formalism to photon-added and depleted coherent states [52], anharmonic oscillator coherent states [53], isotonic oscillator [38, 54] and other nonlinear oscillator systems, and investigating their nonclassical properties is a straightforward matter may be done elsewhere. We hope that the nonlinear charge coherent states and their superpositions have been introduced in the present paper will also find their applications in various physical fields, as well as the standard charge coherent states [16]-[26].

References

- [1] E. Schrödinger, *Naturwissenschaften* 14 (1926) 664.
- [2] R.J. Gluber, *Phys. Rev.* 131 (1963) 2766.
- [3] E.C.G. Sudarshan, *Phys. Rev. Lett.* 10 (1963) 227.
- [4] L.C. Biedenharn, *J. Phys. A* 22 (1989) L873.
- [5] R.L. de Matos Filho, W. Vogel, *Phys. Rev. A* 54 (1996) 4560.
- [6] V.I. Man'ko, G. Marmo, E.C.G. Sudarshan, F. Zaccaria, *Phys. Scrp.* 55 (1997) 528.
- [7] A.O. Barut, L. Girardello, *Commun. Math. Phys.* 21 (1971) 41.
- [8] A.M. Perelomov, *Commun. Math. Phys.* 26 (1972) 222.
- [9] M.M. Nieto, L.M. Simmons, *Phys. Rev. D* 20 (1979) 1321.
- [10] M.M. Nieto, L. M. Simmons, *Phys. Rev. D* 20 (1979) 1332.
- [11] M.M. Nieto, L.M. Simmons, *Phys. Rev. D* 20 (1979) 1342.
- [12] D. Horn, R. Silver, *Phys. Rev. D* 2 (1970) 2082.
- [13] D. Bhaumik, K. Bhaumik, B. Dutta-Roy, *J. Phys. A: Math. Gen.* 9 (1976) 1507.
- [14] S.C. Gou, J. Steinbach, P.L. Knight, *Phys. Rev. A* 54 (1996) 1014.
- [15] G.S. Agarwal, K. Tara, *Phys. Rev. A* 43 (1991) 492.

- [16] M. Bosterli, Phys. Rev. D 16 (1977) 1749.
- [17] K.E. Eriksson, N. Mukunda, B.S. Skagerstam, Phys. Rev. D 24 (1981) 2615.
- [18] P. Chattopadhyay, J. Da Providencia, Nucl. Phys. A 370 (1981) 445.
- [19] M.M. Nieto, Phys. Rev. D 30 (1984) 770.
- [20] B.S. Skagerstam, Phys. Rev. D 22 (1980) 534.
- [21] G.J. Ni, Y.P. Wang, Phys. Rev. D 27 (1983) 969.
- [22] G.S. Agarwal, J. Opt. Soc. Am. B 5 (1988) 1940.
- [23] C.C. Gerry, J. Mod. Opt. 42 (1995) 585.
- [24] S.C. Gou, J. Steibach, P.L. Knight, Phys. Rev. A 54 (1996) R1014.
- [25] S.B. Zheng, G.C. Guo, Quantum. Semiclass. Opt. 10 (1998) 441.
- [26] X.M. Botke, D.J. Scalapino, R.L. Sugar, Phys. Rev. D. 9 (1974) 813.
- [27] D. Horn, R. Silver, Ann. Phys. N.Y. 66 (1971) 509.
- [28] X.-M. Liu, Phys. Lett. A 279 (2001) 123.
- [29] J.Z. Sun, J.S. Wang, C.K. Wang, Phys. Rev. A 44 (1991) 3369.
- [30] X.-M. Liu, Phys. Lett. A 292 (2001) 23.
- [31] S. Chaturvedi, V. Srinivasan, Phys. Rev. A 44 (1991) 8020.
- [32] X.-M. Liu, C. Quesne, Phys. Lett. A 317 (2003) 210.
- [33] L.M. Kuang, F.B. Wang, G.J. Zeng, Phys. Lett. A 179 (1993) 1.
- [34] X.-M. Liu, C. Quesne, F. Song, J. Math. Phys. 46 (2005) 072106.
- [35] Z. Kis, W. Vogel, I. Davidovich, Phys. Rev. A 64 (2001) 033401.
- [36] R. Roknizadeh, M.K. Tavassoly, J. Phys. A: Math. Gen. 37 (2004) 8111.
- [37] R. Roknizadeh, M.K. Tavassoly, J. Math. Phys. 46 (2005) 042110.
- [38] G.R. Honarasa, M.K. Tavassoly, M. Hatami, Opt. Commun. 282 (2009) 2192.
- [39] M.G. Hu and J.L. Chen, Int. J. of Theor. Phys. 46 (2007) 8.
- [40] R. Loudon, P.L. Knight, J. Mod. Opt. 34 (1987) 709.

- [41] K. Wodkiciwicz, J.H. Eberly J. Opt. Soc. Am. B 2 (1985) 458.
- [42] L. Mandel, Opt. Lett. 4 (1979) 205.
- [43] J.P. Antoine, J.R. Gazeau, J.R. Klauder, P. Monceau, K.A. Penson, J. Math. Phys. 42 (2001) 2349.
- [44] J.P. Gazeau, J.R. Klauder, J. Phys. A: Math. Gen. 32 (1999) 123.
- [45] M.K. Tavassoly, and A. Parsaiean, J. Phys. A: Math. Theo. 40 (2007) 9905.
- [46] E.C.G. Sudarshan, Int. J. of Theo. Phys. 32 (1993) 1069.
- [47] S. Twareque Ali, R. Roknizadeh, M.K. Tavassoly, J. Phys. A: Math. Gen. 37 (2004) 4407.
- [48] B. Roy, P. Roy, J. Opt B: Quantum Semiclass. Opt. 2 (2000) 65.
- [49] S. Singh, Phys. Rev. A 25 (1982) 3205.
- [50] A.O. Barut, L. Girardello, Commun. Math. Phys. 21 (1971) 41.
- [51] R. Gilmore, Ann. Phys. (N. Y.) 74 (1972) 391.
- [52] S. Sivakumar J. Phys. A: Math. Gen. 32 (1999) 3441.
- [53] D. Popov, V. Sajfert, I. Zaharie, Physica A: Statistical Mechanics and its Applications, 387 (2008) 4459.
- [54] G.R. Honarasa, M.K. Tavassoly and M. Hatami, Phys. Lett. A 373 (2009) 3931.

FIGURE CAPTIONS

FIG. 1. The plot of $su(1,1)$ -squeezing for x_1 quadrature and x_2 quadrature as a function of $x = |\xi|^2$ for standard charge coherent state ($f(n) = 1$), and nonlinear charge coherent state associated to Pöshl-Teller potential ($f_{PT}(n)$) with fixed parameters $\nu = 3, q = 2$. For the case $f(n) = 1$ the continues curve with point and dot-dashed curve, and for the case $f_{PT}(n)$ the dashed curve and solid curve are respectively indicate x_1 and x_2 $su(1,1)$ -squeezing. Notice that the continues curve with points and dot-dashed curve coincide with each other in the horizontal axis.

FIG. 2. The same as Fig. 1 for even nonlinear charge coherent state associated to standard charge coherent states ($f(n) = 1$) and Pöshl-Teller potential ($f_{PT}(n)$) with fixed parameters $\nu = 3, q = 2$. For the case $f(n) = 1$ the dot-dashed curve and continues curve with points, and for the case $f_{PT}(n)$ the dashed curve and solid curve are respectively indicate x_1 and x_2 $su(1,1)$ -squeezing.

FIG. 3. The same as Fig. 2 except that the X_1 and X_2 quadratures are considered.

FIG. 4. The plot of g -factor as a function of $x = |\xi|^2$ for standard charge coherent state ($f(n) = 1$), and nonlinear charge coherent state associated to Pöshl-Teller potential ($f_{PT}(n)$) with fixed parameters $\nu = 3, q = 2$. The dashed curve corresponds to the case $f(n) = 1$ and the solid curve corresponds to the case $f_{PT}(n)$.

FIG. 5. The plot of G -factor as a function of $x = |\xi|^2$ for standard charge coherent state ($f(n) = 1$), and nonlinear charge coherent state associated to Pöshl-Teller potential ($f_{PT}(n)$) with fixed parameters $\nu = 3, q = 2$. The dashed curve corresponds to the case $f(n) = 1$ and the solid curve corresponds to the case $f_{PT}(n)$.

FIG. 6. The graph of Mandel parameter for the first mode as a function of $x = |\xi|^2$ for standard charge coherent state ($f(n) = 1$), and nonlinear charge coherent state associated to Pöshl-Teller potential ($f_{PT}(n)$) with fixed parameters $\nu = 3, q = 2$. The dashed curve corresponds to the case $f(n) = 1$ and the solid curve corresponds to the case $f_{PT}(n)$.

FIG. 7. The same as Fig. 2 except that standard charge coherent state ($f(n) = 1$), and nonlinear charge coherent state associated to hydrogen-like spectrum ($f_H(n)$) is considered with fixed parameter $q = 2$.

FIG. 8. The same as Fig. 3 except that standard charge coherent state ($f(n) = 1$), and nonlinear charge coherent state associated to hydrogen-like spectrum ($f_H(n)$) is considered with fixed parameter $q = 2$.

FIG. 9. The same as Fig. 4 except that standard odd charge coherent state ($f(n) = 1$), and odd nonlinear charge coherent state associated to hydrogen-like spectrum ($f_H(n)$) is considered with fixed parameter $q = 2$.

FIG. 10. The same as Fig. 6 the graph of generalized Mandel parameter for the second mode as a function of $x = |\xi|^2$ for standard even charge coherent state ($f(n) = 1$), and even nonlinear charge coherent state associated to hydrogen-like spectrum ($f_H(n)$) is considered with fixed parameter $q = 2$.

FIG. 11. The plot of g -factor as a function of $x = |\xi|^2$ for standard charge coherent state ($f(n) = 1$), and nonlinear charge coherent state associated to Barut-Girardello charge coherent states ($f_{BG}(n)$) with fixed parameters $\chi = 1/2, q = 2$. The dashed curve corresponds to the case $f(n) = 1$ and the solid curve corresponds to the case $f_{PT}(n)$.

FIG. 12. The graph of generalized Mandel parameter for the second mode as a function of $x = |\xi|^2$ for standard charge coherent state ($f(n) = 1$), and nonlinear charge coherent state associated to Barut-Girardello ($f_{BG}(n)$) with fixed parameters $\chi = 1/2, q = 2$. The dashed curve corresponds to the case $f(n) = 1$ and the solid curve corresponds to the case $f_{BG}(n)$.